Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, September 2014
(2008 Scheme)
(Special Supplementary)
08.501 : ENGINEERING MATHEMATICS – IV (CMPU)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. A random variable Y is defined as $\cos \pi x$ where X has a uniform p.d.f. over $(-\frac{1}{2}, \frac{1}{2})$ find mean and variance.

2. A continuous random variable X has the distribution function

$$F(x) = \begin{cases} 0, \text{ if } x < 0 \\ K(x-1)^4 \text{ if } 1 < x < 3 \\ 1, \text{ if } x > 3 \end{cases}$$

Find : (a) K (b) the probability density function f(x).

3. If X is normally distributed and the mean of X is 12 and standard deviation is 4. Find :
   1) $P(X \geq 20)$
   2) $P(0 \leq X \leq 12)$.

4. State the principle of least squares.

5. From the following data, find Karl Pearson's coefficient of correlation, $\overline{X} = 5.5$, $\overline{Y} = 4.0$, $\sum X^2 = 385$, $\sum Y^2 = 192$, $\sum (X+Y)^2 = 947$.

6. State Central limit theorem.

7. Define Null hypothesis, Alternative hypothesis, Critical region, Significance level.

8. Solve graphically the L.P.P.

Maximise $Z = 2x_1 + x_2$

Subject to $x_1 + 2x_2 \le 10$,

$x_1 + x_2 \le 6$,

$x_1 - x_2 \le 2$,

$x_1 - 2x_2 \le 1$,

$x_1, x_2 \ge 0$.

9. Obtain all the basic feasible solutions of the system of equations $2x_1 + x_2 - x_3 = 2$, $3x_1 + 2x_2 + x_3 = 3$.

10. Write the dual of the L.P.P.

Maximise $Z = x_1 - x_2 + 3x_3$

Subject to $x_1 + x_2 + x_3 \le 10$,

$2x_1 - x_3 \le 2$,

$2x_1 - 2x_2 + 3x_3 \le 6$

$x_1, x_2, x_3 \ge 0$.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) A random variable X has the density function :

$$f(x) = K, \frac{1}{1+x^2} \text{ in } -\infty < x < \infty$$

$= 0$, otherwise.

Find K and the distribution function F(x). Find $P(X > 0)$.

b) Assume that half of the population is vegetarian so that the chance of an individual being a vegetarian is $\frac{1}{2}$. Assuming that 100 investigators take samples of 10 individual each to see whether they are vegetarian, how many investigators would you expect to report that three people or less were vegetarians ?

c) A manufacturer knows that the condensers he makes contain on the average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 3 or more faulty condensers.

12. a) If X is random variable which follows an exponential distribution with parameter λ with $P(X \leq 1) = P(X > 1)$, find variance of X.

b) If X is a normal variate with mean 2 and variance 4. Y is another normal variate independent of X with mean 2 and variance 3. What is the distribution of X + 2Y ?

c) The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} ne,^{-x/100} & x \geq 0 \\ 0, & x < 0 \end{cases}$$

What is the probability that

a) a computer will function between 50 and 150 has before breaking down ?

b) it will function less than 500 hrs ?

Module – II

13. a) Fit a second degree parabola to the following data :

x :	0	1	2	3	4	5	6
y :	14	18	23	29	36	40	46

b) From the following data, find the most likely value of y when x = 24.

	y	x
Mean	985.8	18.1
S.D.	36.4	2.0

Correlation coefficient is 0.58.

c) Find the coefficient of correlation between price and supply of a commodity from the following data :

Price (in Rs.)	17	18	19	20	21	22	23	24	25	26
Supply (in kg.)	38	37	38	33	32	33	34	29	26	23

14. a) In a random sample of 450 industrial accidents it was found that 230 were due to unsafe working conditions. Construct 95% confidence interval for the corresponding true proportion.

b) In two colleges affiliated to a university 46 out of 200 and 48 out of 250 candidates failed in an examination. If the percentage of failure in the university is 18%, examine whether the colleges differ significantly.

c) On two large populations, there are 30% and 25% respectively fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations ?

Module – III

15. a) A chemical company produces two products A and B. Each unit of product A requires 3 hours on operation I and 4 hours on operation II, while each unit of product B requires 4 hours on operation I and 5 hours on operation II. Total available times for operations I and II are 18 and 21 hours respectively. The production of each unit of product B also results in 3 units of a by-product C at no extra cost. Product A sells at profit of Rs. 3/unit, while B sells at profit of Rs. 8 p.u. unit. By-product C brings a unit profit of Rs. 2 if sold; in case it cannot be sold, the distinction cost in Re. 1/unit. Forecasts indicate that not more than 5 units of C can be sold. Determine the quantities of A and B to be produced, keeping C in mind, so that the profit is maximum. Formulate the problem as L.P.P.

b) Solve the L.P.P. by Simplex method

Maximise $Z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \leq 8,$

$2x_2 + 5x_3 \leq 10,$

$3x_1 + 2x_2 + 4x_3 \leq 15,$

$x_1, x_2, x_3 \geq 0.$

16. a) Solve graphically the L.P.P.

Minimize $Z = 2x_1 + x_2$

Subject to $5x_1 + 10x_2 \leq 50,$

$x_1 + x_2 \geq 1,$

$x_2 \leq 4,$

$4x_1 + 6x_2 \leq 48,$

$x_1, x_2 \geq 0.$

b) Solve the L.P.P. by using dual principle.

Minimize $Z = 2x_1 + 2x_2$

Subject to $2x_1 + 4x_2 \geq 1,$

$x_1 + 2x_2 \geq 1,$

$2x_1 + x_2 \geq 1,$

$x_1, x_2 \geq 0.$